

# Hawking radiation of scalars from accelerating and rotating black holes with NUT parameter

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**Abstract.** We study the quantum tunneling of scalars from charged accelerating and rotating black hole with NUT parameter. For this purpose we use the charged Klein-Gordon equation. We apply WKB approximation and the Hamilton-Jacobi method to solve charged the Klein-Gordon equation. We find the tunneling probability of outgoing charged scalars from the event horizon of this black hole, and hence the Hawking temperature for this black hole.

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## 1 Introduction

Black holes are very subtle and mysterious objects in this universe. In the theory of general relativity, black holes are the solutions of the Einstein field equations. Classically, black holes do not emit any type of radiations and are perfect absorbers. During the start of 1970's, due to the remarkable work of Bakenstein and Stephan Hawking [1–3], the field of black hole physics emerged in a progressive way. The relation between the laws of thermodynamics and black hole thermodynamics [4] is very important in this regard. It is impossible to define a temperature for black holes. Because, every thing goes into the black hole and as a result of this, there is no any output. If this is the case, the second law of thermodynamics would be violated due to entry of matter, which has its own entropy, in to the black hole which results in the decrease of the total entropy of the universe, which contradicts the second law of thermodynamics. Bakenstein first conjectured the relation between the properties of black hole and the laws of thermodynamics. In 1972, again Bakenstein showed that black holes posses entropy,  $S_{bh}$ , similar to the surface area of black hole, whose increase overcomes the decrease of the exterior entropy such that the second law of thermodynamics is preserved. He also related the surface gravity, which is the gravitational acceleration experienced at the surface of the black hole or any object, with temperature of the body in thermal equilibrium. In addition to this, 1974, Stephan Hawking showed that quantum mechanically, black holes actually radiate particles. He also showed that these radiations are purely thermal. Virtual particles and anti particles are permanently created due to vacuum fluctuations at the event horizon of the black hole or near the event horizon. Here, we have three possibilities. Firstly, both the particles may be pulled in to the black hole, secondly, both the particles may go out of the black hole and the third possibility is one particle goes into the black hole and other go away from black hole. For the third possibility, the anti particle must go into the black hole to conserve energy. The particle that has escaped becomes real and appears to be emitted by the black hole. The anti particle, that was pulled into the black hole reduces the black hole mass, charge and the angular momentum. As a result of this antiparticle, the black hole finally shrinks.

Due to this breakthrough in the field of black hole physics, large number of people started to work on these radiations. Some of the important works are given in these references [5–7]. The tunneling method [7, 8, 10–12], first time used by Kraus and Wilczek, is very important and easy method to model the Hawking radiation from black holes. Similarly, Kerner and Mann [13–16] extended this work to large number of black holes and they showed that tunnelling method is very powerful method, which can be applied to variety of black holes. Following the work of Kerner and Mann, recently, this method is applied to higher dimensional

black holes [17–19], black holes in String theory [20], black strings [21–24], accelerating and rotating black holes [25–27], dilaton black holes [28, 29], BTZ black holes [30] and black holes with NUT parameter [13, 31]. In these papers, authors have discussed the emission of fermions and scalar particles from black holes. In this way, large number of modifications are done by different authors [32–34]. For example, the modification of Hawking temperature beyond semiclassical approximation, the use of correct coordinates to model the Hawking radiation and quantum tunneling with back reaction. These Radiations are also discussed for black holes in Yang-Mills theory and Kaluza Klein theory [35, 36]. In this paper, we have applied tunneling method to model the Hawking radiation of scalars from accelerating and rotating black holes with NUT parameter. In section two, we have discussed black holes with NUT parameter. In section three, we have studied the quantum tunneling of charged scalar particles from event horizon of these black configurations. In section four, we have concluded the important results of our work

## 2 Charged accelerating and rotating black holes with NUT parameter

Black holes with NUT parameter [37–40] are very important because they do not satisfy the first law of thermodynamics unless the NUT charge of the space-time vanishes [41] and the presence of NUT charge causes a breakdown of the entropy/area relationship [42, 43]. There are some uncertain and vague notions regarding the physical meaning of the NUT parameter,  $l$ . However, the NUT parameter is related with the gravitomagnetic monopole parameter of the central mass, or a twisting property of the surrounding space-time. The line element for such black hole is given by [39]

$$ds^2 = \frac{-1}{\Omega^2} \left\{ \frac{Q}{\rho^2} \left[ dt - \left( a \sin^2 \theta + 4l \sin \frac{\theta}{2} \right) d\phi \right]^2 - \frac{\rho^2}{Q} dr^2 - \frac{\rho^2}{P} d\theta^2 - \frac{P \sin^2 \theta}{\rho^2} \left[ a dt - (r^2 + (a+l)^2) d\phi \right]^2 \right\}. \quad (2.1)$$

In expanded form, we can write the above line element as

$$ds^2 = \frac{1}{\Omega^2} \left\{ - \left( \frac{Q}{\rho^2} - \frac{a^2 P \sin^2 \theta}{\rho^2} \right) dt^2 + \frac{\rho^2}{Q} dr^2 + \frac{\rho^2}{P} d\theta^2 + \left( \frac{P(r^2 + a^2)^2 \sin^2 \theta}{\rho^2} - \frac{Q a^2 \sin^4 \theta}{\rho^2} \right) d\phi^2 - \frac{2a \sin^2 (P(r^2 + a^2) - Q) dt d\phi}{\rho^2 \Omega^2} \right\}, \quad (2.2)$$

where

$$\begin{aligned}
\Omega &= 1 - \frac{\alpha}{\omega} r (l + \cos \theta), \quad \rho^2 = r^2 + (l + a \cos \theta)^2, \\
P &= 1 - a_3 \cos \theta - a_4 \cos^2 \theta, \\
Q &= \left[ (\omega^2 k + e^2 + g^2) \left( 1 + 2 \frac{\alpha l}{\omega} r \right) - 2Mr + \frac{\omega^2 k}{a^2 - l^2} r^2 \right] \times \\
&\quad \left[ \left( 1 + \frac{\alpha (a - l)}{\omega} r \right) \left( 1 - \frac{\alpha (a + l)}{\omega} r \right) \right], \\
a_3 &= 2 \frac{\alpha a}{\omega} M - 4 \frac{\alpha^2 a l}{\omega^2} (\omega^2 k + e^2 + g^2), \quad a_4 = - \frac{\alpha^2 a^2}{\omega^2} (\omega^2 k + e^2 + g^2) \\
k &= \frac{1 + 2 \frac{\alpha l}{\omega} M - 3 \frac{\alpha^2 l^2}{\omega^2} (e^2 + g^2)}{\frac{\omega^2}{a^2 - l^2} + 3 \alpha^2 l^2}.
\end{aligned}$$

Here, the parameters  $M$ ,  $e$ ,  $g$  and  $\alpha$  represent the mass, electric charge, magnetic charge and acceleration, of the black hole respectively. while  $a$  shows a black hole rotation and  $l$  is a NUT parameter. Now the metric defined by Eq. (2.1) can be written as

$$ds^2 = -f(r, \theta) dt^2 + \frac{dr^2}{g(r, \theta)} + \Sigma(r, \theta) d\theta^2 + K(r, \theta) d\phi^2 - 2H(r, \theta) dt d\phi, \quad (2.3)$$

where  $f(r, \theta)$ ,  $g(r, \theta)$ ,  $\Sigma(r, \theta)$ ,  $K(r, \theta)$ ,  $H(r, \theta)$  are defined below

$$f(r, \theta) = \frac{1}{\Omega^2} \left[ \frac{Q}{\rho^2} - \frac{a^2 P \sin^2 \theta}{\rho^2} \right], \quad g(r, \theta) = \frac{Q \Omega^2}{\rho^2}, \quad \Sigma(r, \theta) = \frac{\rho^2}{P \Omega^2}, \quad (2.4)$$

$$K(r, \theta) = \frac{P \left( r^2 + (a + l)^2 \right)^2 \sin^2 \theta}{\rho^2 \Omega^2} - \frac{Q \left( a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2} \right)^2}{\rho^2 \Omega^2}, \quad (2.5)$$

$$H(r, \theta) = \frac{P a \left( r^2 + (a + l)^2 \right) \sin^2 \theta}{\rho^2 \Omega^2} - \frac{Q \left( a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2} \right)}{\rho^2 \Omega^2}. \quad (2.6)$$

The electromagnetic vector potential for these Black holes is given by [40]

$$\begin{aligned}
A &= \frac{1}{a \left( r^2 + (a \cos \theta + l)^2 \right)} \times \\
&\quad \left[ -er \left( a dt - d\phi \left( (l + a)^2 - (l^2 + a^2 \cos^2 \theta + 2al \cos \theta) \right) \right) \right. \\
&\quad \left. - g(l + a \cos \theta) \left( a dt - d\phi \left( r^2 + (a + l)^2 \right) \right) \right].
\end{aligned} \quad (2.7)$$

The event horizon of the charged accelerating and rotating black hole can be calculated by putting

$$g(r, \theta) = \frac{\Delta(r)}{\Sigma(r, \theta)} = 0, \quad (2.8)$$

where  $\Delta(r) = \frac{Q}{P}$ , So we have the horizon radii

$$r_{\alpha_1} = \frac{\omega}{\alpha(a+l)}, \quad r_{\alpha_2} = \frac{\omega}{\alpha(a-l)}, \quad (2.9)$$

$$r_{\pm} = \frac{a^2 + l^2}{\omega^2 k} \left\{ - \left( (\omega^2 k + e^2 + g^2) \frac{\alpha l}{\omega} - M \right) \pm \sqrt{\left( (\omega^2 k + e^2 + g^2) \frac{\alpha l}{\omega} - M \right)^2 - \frac{\omega^2 k}{a^2 + l^2} (\omega^2 k + e^2 + g^2)} \right\}. \quad (2.10)$$

Here,  $r_{\pm}$  represent the outer horizon and inner horizons respectively. Other two horizons are the acceleration horizons. The angular velocity at outer horizon can be defined as

$$\Omega = \frac{H(r_+, \theta)}{K(r_+, \theta)} = \frac{a}{r_+^2 + (a+l)^2}. \quad (2.11)$$

The line element, Eq. (2.3), can be written also as

$$ds^2 = -F(r, \theta)dt^2 + \frac{dr^2}{g(r, \theta)} + K(r, \theta) \left( d\phi - \frac{2H(r, \theta)}{K(r, \theta)} dt \right)^2 + \Sigma(r, \theta)d\theta^2, \quad (2.12)$$

where  $F(r, \theta)$  is given by

$$F(r, \theta) = f(r, \theta) + \frac{H^2(r, \theta)}{K(r, \theta)}. \quad (2.13)$$

Putting the values of  $f(r, \theta)$ ,  $K(r, \theta)$ ,  $H(r, \theta)$  in above Eq. (2.13), we get

$$F(r, \theta) = \frac{QP\rho^2 \sin^2 \theta}{\Omega^2 \left[ \sin^2 \theta P \left( r^2 + (a+l)^2 \right)^2 - Q \left( a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2} \right)^2 \right]}. \quad (2.14)$$

By using Taylor series, the functions,  $F(r, \theta)$  and  $g(r, \theta)$ , near the event horizon at  $r = r_+$  are given as

$$F(r_+, \theta) = \frac{(r - r_+) Q' (r_+) P \rho^2 \sin^2 \theta}{\Omega^2 \left[ \sin^2 \theta P \left( r_+^2 + (a+l)^2 \right)^2 - (r - r_+) Q' (r_+) \left( a \sin^2 \theta + 4l \sin^2 \frac{\theta}{2} \right)^2 \right]}$$

$$g(r_+, \theta) = \frac{(r - r_+) Q' (r_+) \Omega^2}{\rho^2}.$$

Similarly, we can write the the metric, Eq. (2.12), near the event horizon as

$$ds^2 = F(r_+, \theta)dt^2 + \frac{dr^2}{g(r_+, \theta)} + K(r_+, \theta) \left( d\phi - \frac{H(r_+, \theta)}{K(r_+, \theta)} dt \right)^2 + \Sigma(r_+, \theta)d\theta^2. \quad (2.15)$$

By defining  $d\chi = d\phi - \frac{H(r_+, \theta)}{K(r_+, \theta)} dt$ , we can write the above metric as

$$ds^2 = -F(r_+, \theta)dt^2 + \frac{dr^2}{g(r_+, \theta)} + K(r_+, \theta)d\chi^2 + \Sigma(r_+, \theta)d\theta^2. \quad (2.16)$$

Here, we are following the method of Kerner and Mann, where they used the co rotating frame,  $\chi = \phi - \Omega_H t$ , to deal with quantum tunneling at event horizon at  $r = r_+$ .

### 3 Quantum tunneling

To deal with quantum tunneling near event horizon from charged accelerating and rotating black holes with NUT parameter (Eq. (2.16)) for scalar field  $\Psi$ , we use the charged Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \left( \partial_\mu - \frac{iq}{\hbar} A_\mu \right) \left( \sqrt{-g} g^{\mu\nu} (\partial_\nu - \frac{iq}{\hbar} A_\nu) \Psi \right) - \frac{m^2}{\hbar^2} \Psi = 0, \quad (3.1)$$

where  $A_\mu$  is the electromagnetic potential,  $q$  and  $m$  are the charge and mass of the particle and  $g$  is the determinant of the metric tensor,  $g_{\mu\nu}$ . Here,  $\hbar$  is Planck's constant. In order to apply the WKB approximation, we assume an ansatz of the form

$$\Psi(t, r, \theta, \chi) = e^{\left( \frac{i}{\hbar} I(t, r, \theta, \chi) + I_1(t, r, \theta, \chi) + O(\hbar) \right)}, \quad (3.2)$$

where,  $I$  is the action for outgoing trajectory of the particle. Substituting Eq. (3.2) in Eq. (3.1) and keeping terms only in the lowest order of  $\hbar$  and dividing by exponential term and multiplying by  $\hbar^2$  gives

$$0 = -\frac{(\partial_t I - qA_t)^2}{F(r_+, \theta)} + g(r_+, \theta) (\partial_r I)^2 + \frac{(\partial_\theta I)^2}{\Sigma(r_+, \theta)} + K(r_+, \theta) (\partial_\chi I)^2 + m^2 \quad (3.3)$$

If we look at the symmetries of the space-time, we have the killing vectors,  $\partial_t$  and  $\partial_\chi$ , and we are dealing only with radial trajectories for any  $\theta = \theta_0$ . So there exists a solution for above partial differential equation Eq. (3.3)

$$I = -(E - \Omega_H J)t + W(r, \theta_0) + J\chi. \quad (3.4)$$

Here,  $E$  and  $J$  are constants and are associated with the energy and angular momentum of the particle. If we do not use the co rotating frame then we must use the solution,  $I^* = -Et + W(r, \theta_0) + J\phi$ , but we have used the co rotating frame and replaced  $\phi = \chi + \Omega_H t$  in  $I^*$  and get the solution, Eq. (3.4). After plugging Eq. (3.4) in Eq. (3.3), we get

$$0 = -\frac{(-(E - \Omega_H J) - qA_t)^2}{F(r_+, \theta_0)} + g(r_+, \theta_0) (\partial_r W(r, \theta_0))^2 + K(r_+, \theta_0) J^2 + m^2. \quad (3.5)$$

or

$$W(r) = \pm \int \frac{dr}{\sqrt{g(r_+, \theta_0) F(r_+, \theta_0)}} \times \left( ((E - \Omega_H J) + qA_t)^2 - F(r_+, \theta_0) (K(r_+, \theta_0) J^2 + m^2) \right)^{\frac{1}{2}} \quad (3.6)$$

we have to integrate this integral around the event horizon at  $r = r_+$ . We have simple pole at  $r = r_+$ , so we use residue theory for Semi circle and we get

$$W_\pm(r) = \pm i\pi \frac{r_+^2 + (a+l)^2}{Q'(r_+)} [(E - \Omega_H J) + qA_t], \quad (3.7)$$

where

$$Q'(r_+) = \left( 2(\omega^2 k + e^2 + g^2) \frac{\alpha l}{\omega} - 2M + 2 \frac{\omega^2 k}{a^2 - l^2} r_+ \right) \times \left( 1 + \frac{\alpha(a-l)}{\omega} r_+ \right) \left( 1 - \frac{\alpha(a+l)}{\omega} r_+ \right). \quad (3.8)$$

From Eq. (3.7), we have

$$Im(W_{\pm}) = \pm \pi \frac{r_+^2 + (a+l)^2}{Q'(r_+)} [(E - \Omega_H J) + qA_t]. \quad (3.9)$$

As the probabilities of crossing the horizon from inside to outside and outside to inside is given by [9, 11]

$$P_{out} \propto \exp\left(\frac{-2}{\hbar} Im(I)\right) = \exp\left(\frac{-2}{\hbar} (Im(W_+))\right), \quad (3.10)$$

$$P_{in} \propto \exp\left(\frac{-2}{\hbar} Im(I)\right) = \exp\left(\frac{-2}{\hbar} (Im(W_-))\right). \quad (3.11)$$

Here, + and - indicate for out going and in going particles. From Eq. (3.7), we have

$$W_+ = -W_-. \quad (3.12)$$

This means that the probability of a particle tunneling from inside to outside the horizon is given by

$$\Gamma \propto \frac{P_{out}}{P_{in}} = \exp\left(-\frac{4}{\hbar} Im(W_+)\right). \quad (3.13)$$

or

$$\Gamma = \exp\left(-\frac{4}{\hbar} \pi \frac{r_+^2 + (a+l)^2}{Q'(r_+)} [(E - \Omega_H J) + qA_t]\right). \quad (3.14)$$

From Eq. (3.14), we can say the tunneling probability depends upon the charge,  $q$ , and the term,  $E - \Omega_H J$ , which is the energy of the particle, but not on mass,  $m$ , of the particle. The term,  $-\Omega_H J$ , is due to presence of ergosphere of the black hole. From Eq. (3.14) and [31], we have the same tunneling probability for out going scalars and fermions. By Comparing Eq. (3.14) with Boltzmann factor,  $\Gamma = \exp[-\beta (E - \Omega_H J)]$ , where  $\beta$  is inverse temperature, we have Hawking temperature by choosing  $\hbar = 1$

$$T_H = \frac{Q'(r_+)}{4\pi (r_+^2 + (a+l)^2)}, \quad (3.15)$$

or

$$T_H = \frac{\left((\omega^2 k + e^2 + g^2) \frac{\alpha l}{\omega} - M + \frac{\omega^2 k}{a^2 - l^2} r_+\right) \left(1 + \frac{\alpha(a-l)}{\omega} r_+\right) \left(1 - \frac{\alpha(a+l)}{\omega} r_+\right)}{2\pi (r_+^2 + (a+l)^2)}, \quad (3.16)$$

which is consistent with previous literature [31] and this temperature is sufficiently general and reduces to all the special cases after plugging the other parameters equal to zero.

It is pertinent to mention here that Eq. (3.6) is not canonically invariant so it is not good physical observer [47]. One can write the canonical transformation to rectify Eq. (3.6). Initially, It is shown that the tunneling probability only depends upon imaginary part of the action coming from spatial coordinates but there is contribution of imaginary part of action coming from temporal component to the tunneling probability [48–50]. Due to this temporal contribution, the problem of so called factor two [47] to Hawking radiations is solved.

## 4 Conclusion

In this paper, we have studied the Hawking radiation of charged scalar particles from charged accelerating and rotating black holes with NUT parameter. By using the Hamilton-Jacobi method we have solved the charged Klein-Gordon equation. For this purpose, we have employed the WKB approximation to charged Klein-Gordon equation to derive the tunneling probability of outgoing scalar particles. At the end, by comparing with the Boltzmann factor of energy for particle, we have derived the Hawking temperature for these black holes. These results are found to be consistent with the previous literature. When  $l = 0$ ,  $k = 1$  in Eq. (3.16), we recover the Hawking temperature of the accelerating and rotating black holes with electric and magnetic charges [25–27]. When  $\alpha = 0$ , the Hawking temperature of the non-accelerating black holes is recovered [44]. For  $l = 0$ ,  $k = 1$ ,  $\alpha = 0$ , the Hawking temperature of the Kerr-Newman black hole [16] is obtained which is further reduced to the temperature of the RN black hole [45] (for  $a = 0$ ). In the absence of charge, the temperature exactly reduces to the Hawking temperature of the Schwarzschild black hole [46].

In this way we have studied the tunneling probability of out going scalars from the event horizon of the charged accelerating and rotating black holes with NUT parameter. The fermions and scalars tunnels out from event horizon at the same rate and tunneling probability of both scalars and fermions depends upon the charge and energy of the particles but not on the mass of particles.

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